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REMARKS ON SOME LIMITS APPEARING IN THE THEORY OF ALMOST PERIODIC FUNCTIONS

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Abstract. In this note we are going to present new short proofs concerning either the existence or the non-existence of some limits appearing in the theory of almost periodic functions. Our proofs are completely different from those presented in the papers [1] and [3].

1. Introduction

In the rich theory of almost periodic functions (see e.g. [6]) problems concerning the evaluation of the limit

$$\lim_{x\to+\infty}\frac{f(x)}{2+\cos x+\cos(x\sqrt{2})}, \tag{1.1}$$

where $f\colon \mathbb{R}\to\mathbb{R}$ is an exponential function or a polynomial (cf. [1] or [3]), quite frequently appear. This is connected to the fact that the function

$$x\mapsto \frac{1}{2+\cos x+\cos(x\sqrt{2})}\quad \text{for } x\in\mathbb{R}$$

constitutes a classical example of a function which is either almost periodic in the sense of Levitan (briefly: LAP) or almost periodic with respect to the Lebesgue measure (briefly: μ.a.p.) (see e.g. [4]). In particular, in [1] the authors used the theory of continued fractions to prove that the limit (1.1) is equal to zero if

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