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NODAL SOLUTION FOR A PLANAR PROBLEM WITH FAST INCREASING WEIGHTS

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Abstract. In this paper we prove the existence of a sign-changing solutions for the equation

$$-\Delta u - \frac{1}{2}(x \cdot \nabla u) = f(u), \quad x \in \mathbb{R}^2,$$

where f has exponential critical growth in the sense of the Trudinger–Moser inequality. In the proof we apply variational methods.

1. Introduction

Consider the nonlinear heat equation

$$v_t - \Delta v = |v|^{p-1}v \quad \text{on } (0,\infty) \times \mathbb{R}^N.$$

If we try to find solutions of the form $v(t,x) = t^{-1/(p-1)}u(t^{-1/2}x)$, a straightforward calculation shows that $u\colon \mathbb{R}^N \to \mathbb{R}$ needs to satisfy

$$-\Delta u - \frac{1}{2}(x \cdot \nabla u) = \frac{1}{p-1}\, u + |u|^{p-1}u \quad \text{in } \mathbb{R}^N.$$

Solutions v with the above profile are called self-similar solutions (see [15], and [8]). Besides providing qualitative properties like global existence, blow-up

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