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THE EFFECT OF TOPOLOGY ON THE NUMBER OF POSITIVE SOLUTIONS OF ELLIPTIC EQUATION INVOLVING HARDY–LITTLEWOOD–SOBOLEV CRITICAL EXPONENT

Divya Goel

Abstract. In this article we are concerned with the following Choquard equation

$$-\Delta u = \lambda|u|^{q-2}u + \Big(\int_\Omega \frac{|u(y)|^{2^*_\mu}}{|x-y|^\mu}\,dy\Big)|u|^{2^*_\mu-2}u, \quad u>0, \quad \text{in } \Omega,$$
$$u = 0 \qquad \text{on } \partial\Omega,$$

where Ω is an open bounded set with continuous boundary in $\mathbb{R}^N (N \geq 3)$, $2^*_\mu = (2N-\mu)/(N-2)$ and $q \in [2, 2^*)$ where $2^* = 2N/(N-2)$. Using Lusternik–Schnirelman theory, we associate the number of positive solutions of the above problem with the topology of Ω. Indeed, we prove that if $\lambda < \lambda_1$, then problem has $\mathrm{cat}_\Omega(\Omega)$ positive solutions whenever $q \in [2, 2^*)$ and $N > 3$ or $4 < q < 6$ and $N = 3$.

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