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EFFECT OF EXTERNAL POTENTIALS IN A COUPLED SYSTEM OF MULTI-COMPONENT INCONGRUENT DIFFUSION

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Abstract. This work is devoted to investigations of some interesting aspects of a multi-component Reaction-Diffusion system of the form

$$\partial_t z = \mathbf{D}\Delta_x z + M(x)z + W(x)|z|^{p-2}\beta z, \quad z\colon \mathbb{R}\times\mathbb{R}^N \to \mathbb{R}^{2K},\ N \geq 2$$

where M, W are external potential functions, $\mathbf{D}$ and β are matrices of diffusion coefficients and coupling constants respectively. When the diffusion rate is small, we show that the geometric shapes of the external potential functions will influence the multiplicity of solutions to the system. It is also of interest to know that, for $z = (u, v)$, we shall deal with standard diffusion coefficients $D_u > 0$ and the incongruent diffusion coefficients $D_v < 0$ which has generally been overlooked in the study of Reaction–Diffusion systems.

1. Introduction

1.1. Some backgrounds and previous results. A system of Reaction–Diffusion (RD) equations comprises of reaction terms and diffusion terms, i.e. the typical form is as follows:

$$\partial_t z = \operatorname{div}_x\big(\mathbf{D}\nabla_x z\big) + M(x)z + f(x, z) \tag{1.1}$$

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