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SUBSPACES OF INTERVAL MAPS RELATED TO THE TOPOLOGICAL ENTROPY

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Abstract. For $a \in [0, +\infty)$, the function space $E_{\geq a}$ ($E_{>a}$; $E_{\leq a}$; $E_{<a}$) of all continuous maps from $[0, 1]$ to itself whose topological entropies are larger than or equal to a (larger than a; smaller than or equal to a; smaller than a) with the supremum metric is investigated. It is shown that the spaces $E_{\geq a}$ and $E_{>a}$ are homeomorphic to the Hilbert space l_2 and the spaces $E_{\leq a}$ and $E_{<a}$ are contractible. Moreover, the subspaces of $E_{\leq a}$ and $E_{<a}$ consisting of all piecewise monotone maps are homotopy dense in them, respectively.

1. Introduction

One of the central topics in the study of infinite-dimensional topology is the problem which function spaces are homeomorphic to the separable infinite dimensional Hilbert space l_2 or its well-behaved subspaces. The well-known Anderson–Kadec's theorem states that the countable infinite product $\mathbb{R}^{\mathbb{N}}$ of lines is homeomorphic to l_2, see [1], [10]. Using this result, it was proved that the space of real valued maps of an infinite compact metric space with the supremum metric is homeomorphic to l_2. See [4], [14], [15] for more on this topic. Moreover, in [6], the authors proved that the function space of real valued maps of an

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