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THE WEAK FIXED POINT PROPERTY OF DIRECT SUMS OF SOME BANACH SPACES

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Abstract. We prove that if a Banach space X has the weak fixed point property and Y satisfies the condition $M(Y) > 1$, then the direct sum $X \oplus Y$ with a uniformly convex norm has the weak fixed point property.

1. Introduction

A Banach space X has the fixed point property if for every nonempty closed convex and bounded set K every nonexpansive mapping $T\colon K \to K$, i.e. a mapping such that $\|Tx-Ty\| \leq \|x-y\|$ for all $x, y \in K$, has a fixed point. Similarly, the space X has the weak fixed point property if for every nonempty weakly compact convex set K every nonexpansive mapping $T\colon K \to K$ has a fixed point. In 1965 Browder [4] proved that every uniformly convex Banach space has the fixed point property. Since then, many papers about geometric conditions of a space implying the fixed point property have been published. In 1996 Domínguez Benavides [8] introduced the coefficient $M(X)$ of a Banach space X and proved that if $M(X) > 1$, then X has the weak fixed point property. Using this result García Falset, Llorens Fuster and Mazcuñan Navarro [9] solved a long-standing problem: every uniformly nonsquare space has the fixed point property.

One of research directions in the fixed point theory is to study conditions under which a direct sum of spaces has the fixed point property. The simplest case is when a geometric property, which implies the fixed point property, is

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