FIXED POINT INDEX THEORY
FOR PERTURBATION OF EXPANSIVE MAPPINGS
BY k-SET CONTRACTIONS

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Abstract. In this work, we develop a fixed point index theory for the sum of k-set contractions and expansive mappings with constant $h > 1$ when $0 \leq k < h-1$ as well as in the limit case $k = h-1$. After computing this new index, several fixed point theorems and recent results are derived, including Krasnosel'skii type theorems. Two examples of application illustrate the theoretical results.

1. Introduction

Starting from the Krasnosel'skiĭ fixed point theorem (KFPT for short) [22], the fixed point theory for sums of operators developed promptly and has been widely extended to various types of nonlinear mappings (see, e.g. [10], [29], [36]) in theory as well as in applications to many problems in nonlinear sciences. KFPT (1958) concerns the sum of a contraction and a compact mapping and turns out to be a generalization of Banach's contraction mapping principle (1922) and Schauder's fixed point theorem (1930) [33]. However, its proof uses both of these important results. It states that the sum $T + F$ has at least one fixed point in D whenever the mappings $T, F \colon D \to E$ satisfy the following conditions:

(a) for all $x, y \in D$, $T(x) + F(y) \in D$.
(b) T is a contraction.