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ON THE EXISTENCE OF SKYRMIONS IN PLANAR LIQUID CRYSTALS

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Abstract. The study of topologically nontrivial field configurations is an important topic in many branches of physics and applied sciences. In this paper we are interested to the existence of such structures, the so-called skyrmions, in the context of liquid crystals. More precisely, we consider a two-dimensional nematic or cholesteric liquid crystal. In the nematic case we use a Bogomol'nyi type decomposition in order to get a topological lower bound on the configurations with a given degree for the full Oseen–Frank energy functional, and so we can find a global minimum of degree ± 1 for the energy. Then we consider the cholesteric case in presence of an electric field under the one constant approximation assumption, and, by using the concentration-compactness method, we prove the existence of a minimum again on the configurations of degree ± 1, for sufficiently large electric fields.

1. Introduction

Let us consider a thin infinity plate of a nematic or cholesteric liquid crystal, possibly in the presence of an orthogonal applied electric (or magnetic) field $\mathbf{E}$. In the Oseen–Frank model, the configurations of the liquid crystal are described by means of functions $u\colon \mathbb{R}^2 \to S^2$, where S^2 is the unit sphere of $\mathbb{R}^3$, and the unit length vector $u(x)$ is the the optical axis, namely the (average) direction of the molecules at the point $x \in \mathbb{R}^2$. The energy density is given by $2W = K_1 \operatorname{div}(u)^2 + K_2(u \cdot \operatorname{curl} u + \tau)^2 + K_3|u \times \operatorname{curl} u|^2 - \varepsilon_a(\mathbf{E} \cdot u)^2$, where K_1, K_2

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