

MULTIPLE SOLUTIONS FOR SCHRÖDINGER–POISSON SYSTEMS WITH CRITICAL NONLOCAL TERM

ZUJI GUO

ABSTRACT. This paper is concerned with the existence of positive bound state solutions for Schrödinger–Poisson systems with critical nonlocal term:

$$(P) \quad \begin{cases} -\Delta u = \phi|u|^3u + \lambda Q(x)|u|^{q-2}u & \text{in } \mathbb{R}^3, \\ -\Delta \phi = |u|^5 & \text{in } \mathbb{R}^3. \end{cases}$$

Under certain assumptions on Q and λ , we prove that (P) has multiple positive bound state solutions by decomposition the Nehari manifold and fine estimates.

1. Introduction and main results

In the last two decades the following Schrödinger–Poisson systems

$$(1.1) \quad \begin{cases} -\Delta u + V(x)u + \phi u = |u|^{q-2}u & \text{in } \mathbb{R}^3, \\ -\Delta \phi = \varepsilon|u|^2 & \text{in } \mathbb{R}^3, \end{cases}$$

have been intensively studied by a lot of researchers, due to the fact that solutions $(u(x), \phi(x))$ of (1.1) correspond to standing wave solutions $(e^{-i\lambda t}u(x), \phi(x))$ of

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