MULTIPLE SOLUTIONS
FOR SCHRÖDINGER–POISSON SYSTEMS
WITH CRITICAL NONLOCAL TERM

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ABSTRACT. This paper is concerned with the existence of positive bound state solutions for Schrödinger–Poisson systems with critical nonlocal term:

$$(\mathcal{P}) \qquad \begin{cases} -\Delta u = \phi|u|^3 u + \lambda Q(x)|u|^{q-2}u & \text{in } \mathbb{R}^3, \\ -\Delta \phi = |u|^5 & \text{in } \mathbb{R}^3. \end{cases}$$

Under certain assumptions on Q and λ, we prove that $(\mathcal{P})$ has multiple positive bound state solutions by decomposition the Nehari manifold and fine estimates.

1. Introduction and main results

In the last two decades the following Schrödinger–Poisson systems

$$(1.1) \qquad \begin{cases} -\Delta u + V(x)u + \phi u = |u|^{q-2}u & \text{in } \mathbb{R}^3, \\ -\Delta \phi = \varepsilon|u|^2 & \text{in } \mathbb{R}^3, \end{cases}$$

have been intensively studied by a lot of researchers, due to the fact that solutions $(u(x), \phi(x))$ of (1.1) correspond to standing wave solutions $(e^{-i\lambda t}u(x), \phi(x))$ of