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SYMMETRIC TOPOLOGICAL COMPLEXITY FOR FINITE SPACES AND CLASSIFYING SPACES

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Abstract. We present a combinatorial approach to the symmetric motion planning in polyhedra using finite spaces. For a finite space P and a positive integer k, we introduce two types of combinatorial invariants, $\mathrm{CC}^S_k(P)$ and $\mathrm{CC}^\Sigma_k(P)$, that are closely related to the design of symmetric robotic motions in the k-iterated barycentric subdivision of the associated simplicial complex $\mathcal{K}(P)$. For the geometric realization $\mathcal{B}(P) = |\mathcal{K}(P)|$, we show that the first $\mathrm{CC}^S_k(P)$ converges to Farber–Grant's symmetric topological complexity $\mathrm{TC}^S(\mathcal{B}(P))$ and the second $\mathrm{CC}^\Sigma_k(P)$ converges to Basabe–González–Rudyak–Tamaki's symmetrized topological complexity $\mathrm{TC}^\Sigma(\mathcal{B}(P))$ as k becomes larger.

1. Introduction

The *topological complexity* $\mathrm{TC}(X)$ of a space X is a homotopy invariant introduced by Farber [8] to study the robotic motion planning in X. For a positive integer $n \geq 1$, the equality $\mathrm{TC}(X) = n$ implies that we need at least n local motion planning rules on open sets covering X to design continuous robotic motions in X. Farber and Grant considered additional practical motion planning rules [9] by taking symmetricity into account, and extended TC to the *symmetric topological complexity* TC^S. Another symmetrization TC^Σ of the topological

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