

## NONLOCAL SCHRÖDINGER EQUATIONS FOR INTEGRO-DIFFERENTIAL OPERATORS WITH MEASURABLE KERNELS

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ABSTRACT. In this paper we investigate the existence of positive solutions for the problem

$$-\mathcal{L}_K u + V(x)u = f(u)$$

in  $\mathbb{R}^N$ , where  $-\mathcal{L}_K$  is an integro-differential operator with measurable kernel  $K$ . Under appropriate hypotheses, we prove by variational methods that this equation has a nonnegative solution.

### 1. Introduction

In this paper we consider the class of integro-differential Schrödinger equations

$$(P) \quad -\mathcal{L}_K u + V(x)u = f(u) \quad \text{in } \mathbb{R}^N,$$

where  $-\mathcal{L}_K$  is an integro-differential operator, given by

$$-\mathcal{L}_K u(x) = 2 \lim_{\varepsilon \rightarrow 0^+} \int_{|x-y|>\varepsilon} (u(y) - u(x))K(x-y) dy$$

and  $K$  satisfies general properties. This study leads both to nonlocal and to nonlinear difficulties. For example, we cannot benefit from the  $s$ -harmonic extension of Caffarelli and Silvestre (see [11]) or commutator properties (see [29]).

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