Topological Methods in Nonlinear Analysis
Volume 1, No. 54, 2019, 345–370
DOI: 10.12775/TMNA.2019.049

© 2019 Juliusz Schauder Centre for Nonlinear Studies
Nicolaus Copernicus University in Toruń

A GLOBAL MULTIPLICITY RESULT FOR A VERY SINGULAR CRITICAL NONLOCAL EQUATION

Jacques Giacomoni — Tuhina Mukherjee — Konijeti Sreenadh

Abstract. In this article we show the global multiplicity result for the following nonlocal singular problem

$$(\mathrm{P}_\lambda) \qquad (-\Delta)^s u = u^{-q} + \lambda u^{2_s^*-1}, \quad u>0 \quad \text{in } \Omega, \quad u=0 \quad \text{in } \mathbb{R}^n \setminus \Omega,$$

where Ω is a bounded domain in $\mathbb{R}^n$ with smooth boundary $\partial\Omega$, $n>2s$, $s\in(0,1)$, $\lambda>0$, $q>0$ satisfies $q(2s-1)<(2s+1)$ and $2_s^*=2n/(n-2s)$. Employing the variational method, we show the existence of at least two distinct weak positive solutions for (P_λ) in X_0 when $\lambda\in(0,\Lambda)$ and no solution when $\lambda>\Lambda$, where $\Lambda>0$ is appropriately chosen. We also prove a result of independent interest that any weak solution to (P_λ) is in $C^\alpha(\mathbb{R}^n)$ with $\alpha=\alpha(s,q)\in(0,1)$. The asymptotic behaviour of weak solutions reveals that this result is sharp.

1. Introduction

In this article we prove the existence, multiplicity and Hölder regularity of weak solutions to the following fractional critical and singular elliptic equation

$$(\mathrm{P}_\lambda) \qquad (-\Delta)^s u = u^{-q} + \lambda u^{2_s^*-1}, \quad u>0 \quad \text{in } \Omega, \quad u=0 \quad \text{in } \mathbb{R}^n \setminus \Omega,$$

where Ω is a bounded domain in $\mathbb{R}^n$ with smooth boundary $\partial\Omega$, $n>2s$, $s\in(0,1)$, $\lambda>0$, $q>0$ satisfies $q(2s-1)<(2s+1)$ and $2_s^*=2n/(n-2s)$. The fractional

2010 *Mathematics Subject Classification.* 35R11, 35R09, 35A15.
Key words and phrases. Fractional Laplacian; very singular nonlinearity; variational method; Hölder regularity.