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MULTIPLE NORMALIZED SOLUTIONS FOR CHOQUARD EQUATIONS INVOLVING KIRCHHOFF TYPE PERTURBATION

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Abstract. In this paper we study the existence of critical points of the C^1 functional

$$E(u) = \frac{a}{2}\int_{\mathbb{R}^N} |\nabla u|^2\,dx + \frac{b}{4}\Big(\int_{\mathbb{R}^N} |\nabla u|^2\,dx\Big)^2 - \frac{1}{2p}\int_{\mathbb{R}^N} (I_\alpha * |u|^p)|u|^p\,dx$$

under the constraint

$$S_c = \Big\{u \in H^1(\mathbb{R}^N) \;\Big|\; \int_{\mathbb{R}^N} |u|^2\,dx = c^2\Big\},$$

where $a > 0$, $b > 0$, $N \geq 3$, $\alpha \in (0, N)$, $(N+\alpha)/N < p < (N+\alpha)/(N-2)$ and I_α is the Riesz Potential. When p belongs to different ranges, we obtain the threshold values separating the existence and nonexistence of critical points of E on S_c. We also study the behaviors of the Lagrange multipliers and the energies corresponding to the constrained critical points when $c \to 0$ and $c \to +\infty$, respectively.

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