Topological Methods in Nonlinear Analysis
Volume 54, No. 1, 2019, 257–274
DOI: 10.12775/TMNA.2019.041

NONLINEAR VECTOR DUFFING INCLUSIONS WITH NO GROWTH RESTRICTION ON THE ORIENTOR FIELD

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Abstract. We consider nonlinear multivalued Dirichlet Duffing systems. We do not impose any growth condition on the multivalued perturbation. Using tools from the theory of nonlinear operators of monotone type, we prove existence theorems for the convex and the nonconvex problems. Also we show the existence of extremal trajectories and show that such solutions are $C_0^1(T,\mathbb{R}^N)$-dense in the solution set of the convex problem (strong relaxation theorem).

1. Introduction

In this paper, we continue our work on multivalued nonlinear Duffing systems initiated in Papageorgiou–Vetro–Vetro [14]. So, the system under consideration is the following:

$$\begin{cases} -a(u'(t))' - r(t)|u'(t)|^{p-2}u'(t) \in F(t,u(t)) \quad \text{for a.a. } t \in T = [0,b], \\ u(0) = u(b) = 0, \quad 1 < p < +\infty. \end{cases} \tag{1.1}$$

Here $a\colon \mathbb{R}^N \to \mathbb{R}^N$ is a monotone homeomorphism and incorporates as special cases many differential operators of interest. In [14] we proved existence theorems for both the convex and nonconvex problems (that is, F is convex valued and respectively nonconvex valued). Also, we proved a relaxation theorem showing

2010 *Mathematics Subject Classification.* Primary: 34A60; Secondary: 34B15.
Key words and phrases. Duffing system; extremal solutions; nonlinear operator of monotone type; strong relaxations.