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THE CONTINUITY OF ADDITIVE AND CONVEX FUNCTIONS WHICH ARE UPPER BOUNDED ON NON-FLAT CONTINUA IN $\mathbb{R}^n$

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Abstract. We prove that for a continuum $K \subset \mathbb{R}^n$ the sum K^{+n} of n copies of K has non-empty interior in $\mathbb{R}^n$ if and only if K is not flat in the sense that the affine hull of K coincides with $\mathbb{R}^n$. Moreover, if K is locally connected and each non-empty open subset of K is not flat, then for any (analytic) non-meager subset $A \subset K$ the sum A^{+n} of n copies of A is not meager in $\mathbb{R}^n$ (and then the sum A^{+2n} of $2n$ copies of the analytic set A has non-empty interior in $\mathbb{R}^n$ and the set $(A-A)^{+n}$ is a neighbourhood of zero in $\mathbb{R}^n$). This implies that a mid-convex function $f\colon D \to \mathbb{R}$ defined on an open convex subset $D \subset \mathbb{R}^n$ is continuous if it is upper bounded on some non-flat continuum in D or on a non-meager analytic subset of a locally connected nowhere flat subset of D.

1. Introduction

Let X be a linear topological space over the field of real numbers. A function $f\colon X \to \mathbb{R}$ is called *additive* if $f(x+y) = f(x) + f(y)$ for all $x, y \in X$.

A function $f\colon D \to \mathbb{R}$ defined on a convex subset $D \subset X$ is called *mid-convex* if $f((x+y)/2) \le (f(x)+f(y))/2$ for all $x, y \in D$.

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