INFINITELY MANY SOLUTIONS
FOR A CLASS OF CRITICAL CHOQUARD EQUATION
WITH ZERO MASS

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ABSTRACT. In this paper we investigate the following nonlinear Choquard equation

$$-\Delta u = \left(\int_{\mathbb{R}^N} \frac{G(y,u)}{|x-y|^\mu} \, dy \right) g(x,u) \quad \text{in } \mathbb{R}^N,$$

where $0 < \mu < N$, $N \geq 3$, $g(x,u)$ is of critical growth in the sense of the Hardy–Littlewood–Sobolev inequality and $G(x,u) = \int_0^u g(x,s) \, ds$. By applying minimax procedure and perturbation technique, we obtain the existence of infinitely many solutions.

1. Introduction and main results

The aim of the present paper is to consider the following nonlinear critical Choquard equation with a subcritical nonlocal term

(1.1)
$$
\begin{cases}
-\Delta u = \left(\displaystyle\int_{\mathbb{R}^N} \frac{\delta|u(y)|^{2^*_\mu} + \lambda K(y)|u(y)|^p}{|x-y|^\mu} \, dy \right) \\
\qquad\qquad \left(\delta|u|^{2^*_\mu - 2} u + \dfrac{p}{2^*_\mu} \lambda K(x)|u|^{p-2} u \right) \quad \text{in } \mathbb{R}^N, \\
u \in D^{1,2}(\mathbb{R}^N),
\end{cases}
$$