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# TOPOLOGICAL CHARACTERISTICS OF SOLUTION SETS FOR FRACTIONAL EVOLUTION EQUATIONS AND APPLICATIONS TO CONTROL SYSTEMS

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Abstract. This paper explores an abstract Riemann–Liouville fractional evolution model with a weighted delay initial condition. We develop the resolvent technique, a generalization of semigroup method, to formulate an appropriate notion of mild solutions to this abstract system and present the topological characteristics of the corresponding solution set in a weighted space. Furthermore, in view of the topological characteristics, we analyze the approximate controllability of the abstract system without Lipschitz assumption. We end up addressing an infinite dimensional fractional delay diffusion control system and a finite dimensional fractional ordinary differential control system by utilizing our theoretical findings.

## 1. Introduction

Fractional differential systems have in recent years been active research topics because of their broad applicability in describing many physical problems with memory features and genetic properties. Many fruitful findings on fractional equations have been reported in the literature (see [23], [29], [33], [36]).

Du and Wang [8] demonstrated that Riemann–Liouville fractional systems are more suitable to describe some practical applications in viscoelastic materials

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