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A WEIGHTED TRUDINGER–MOSER TYPE INEQUALITY AND ITS APPLICATIONS TO QUASILINEAR ELLIPTIC PROBLEMS WITH CRITICAL GROWTH IN THE WHOLE EUCLIDEAN SPACE

Francisco S.B. Albuquerque — Sami Aouaoui

Abstract. We establish a version of the Trudinger–Moser inequality involving unbounded or decaying radial weights in weighted Sobolev spaces. In the light of this inequality and using a minimax procedure we also study existence of solutions for a class of quasilinear elliptic problems involving exponential critical growth.

1. Introduction and main results

We recall that if Ω is a bounded domain in $\mathbb{R}^n$ $(n \geq 2)$, the classical Trudinger–Moser inequality (cf. [31], [38]) asserts that $e^{\alpha|u|^{n'}} \in L^1(\Omega)$, for all $u \in W_0^{1,n}(\Omega)$ and $\alpha > 0$ and there exists a constant $C(n) > 0$ such that

$$\sup_{\|u\|_n \leq 1} \int_\Omega e^{\alpha|u|^{n'}}\, dx \leq C(n)|\Omega|, \quad \text{if } \alpha \leq \alpha_n, \tag{1.1}$$

where $n' = n/(n-1)$, $\alpha_n = n\omega_{n-1}^{1/(n-1)}$, $\|u\|_n := \big(\int_\Omega |\nabla u|^n\, dx\big)^{1/n}$ and ω_{n-1} is the surface area of the unit sphere in $\mathbb{R}^n$. Moreover, the inequality (1.1)

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