

CLASSIFICATION OF RADIAL SOLUTIONS TO HÉNON TYPE EQUATION ON THE HYPERBOLIC SPACE

SHOICHI HASEGAWA

ABSTRACT. We devote this paper to classifying radial solutions of a weighted semilinear elliptic equation on the hyperbolic space. More precisely, for a weighted Lane–Emden equation on the hyperbolic space, we shall study the sign and asymptotic behavior of the radial solutions. We shall also show the existence of fast-decay sign-changing solutions to the Lane–Emden equation on the hyperbolic space.

1. Introduction

In this paper, we shall investigate the structure of radial solutions to the following weighted semilinear elliptic equation:

$$(H) \quad -\Delta_g u = (\sinh r)^\alpha |u|^{p-1} u \quad \text{in } \mathbb{H}^N,$$

where $N \geq 2$, $p > 1$, and $\alpha > -2$. Here, \mathbb{H}^N denotes the N -dimensional hyperbolic space in terms of the spherical coordinates, $r > 0$ represents the geodesic distance on \mathbb{H}^N , and Δ_g denotes the Laplace–Beltrami operator on \mathbb{H}^N .

The structure of radial solutions to semilinear elliptic equations has attracted a great interest. In particular, the following Hénon type equation has been well

2010 *Mathematics Subject Classification*. Primary: 58J05; Secondary: 35B05, 58K55.

Key words and phrases. Semilinear elliptic equation; decay rate; sign-changing solutions.

The author was supported by Research Fellow of Japan Society for the Promotion of Science (No. 16J01320).