ON THE TOPOLOGICAL DEGREE OF PLANAR MAPS
AVOIDING NORMAL CONES

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ABSTRACT. The classical Poincaré–Bohl theorem provides the existence of a zero for a function avoiding external rays. When the domain is convex, the same holds true when avoiding normal cones. We consider here the possibility of dealing with nonconvex sets having inward corners or cusps, in which cases the normal cone vanishes. This allows us to deal with situations where the topological degree may be strictly greater than 1.

1. Introduction

Let $\Omega$ be an open and bounded planar set, whose boundary $\partial \Omega$ is a Jordan curve, and let $f : \Omega \to \mathbb{R}^2$ be a continuous function such that $0 \notin f(\partial \Omega)$. The aim of this paper is to provide some conditions on the behaviour of the function at the boundary which guarantee that the Brouwer topological degree $\deg(f, \Omega)$ is a positive number. It is well known that, in such a case, there will be some $x \in \Omega$ such that $f(x) = 0$ (sometimes called equilibrium).

In the case when $\Omega$ is convex, the normal cone at a given point $\bar{x} \in \partial \Omega$ is defined as

$$\mathcal{N}_\Omega(\bar{x}) = \{v \in \mathbb{R}^2 : \langle v, x - \bar{x} \rangle \leq 0, \text{ for every } x \in \Omega\}.$$ 

Here, as usual, $\langle \cdot, \cdot \rangle$ denotes the euclidean scalar product in $\mathbb{R}^2$, with associated norm $\| \cdot \|$. Let us recall the following known result.

2010 Mathematics Subject Classification. 47H10.

Key words and phrases. Poincaré–Bohl; topological degree; avoiding cones condition.