FORMAL BARYCENTER SPACES WITH WEIGHTS: 
THE EULER CHARACTERISTIC

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ABSTRACT. We compute the Euler characteristic with compact supports $\chi_c$ of the formal barycenter spaces with weights of some locally compact spaces, connected or not. This reduces to the topological Euler characteristic $\chi$ when the weights of the singular points are less than one. As foresighted by Andrea Malchiodi, our formula is related to the Leray–Schauder degree for mean field equations on a compact Riemann surface obtained by C.C. Chen and C.S. Lin.

1. Statement of the main result

Given a space $X$, we will write $B_k(X)$ for the space of formal barycenters of $k$ points in $X$ [11]. By construction there are inclusions $B_k(X) \hookrightarrow B_{k+1}(X)$ for all $k$ and we will write $B(X)$ the direct limit. This is known to be a contractible space if $X$ is of the homotopy type of a CW.

Let $Q_r := \{y_1, \ldots, y_r\} \subset X$ be a fixed finite set of “singular points” in $X$. We assign to every $x \in X$ a weight

$$w(x) = \begin{cases} 1 & \text{if } x \notin Q_r, \\ w_i & \text{if } x = y_i, \end{cases}$$

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