GLOBAL SECONDARY BIFURCATION, SYMMETRY BREAKING AND PERIOD-DOUBLING

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ABSTRACT. In this paper we provide a criterion for global secondary bifurcation via symmetry breaking. As an application, the occurrence of period-doubling bifurcations for the Lugiato-Lefever equation is proved.

1. Introduction

The aim of this paper is to provide a sufficient condition for global secondary bifurcation via symmetry breaking for equations of the form

\[ F(x, \lambda) = 0, \]

where \( x \in X \) belongs to a Banach space and \( \lambda \in \mathbb{R} \) is a real parameter. Bifurcation theory is about finding solutions near a given family of trivial solutions of (1.1). For instance, if \( F(0, \lambda) = 0 \) for all \( \lambda \in \mathbb{R} \), then the trivial solution family is given by \( \{(0, \lambda) : \lambda \in \mathbb{R}\} \subset X \times \mathbb{R} \). More generally, if \( T \subset X \times \mathbb{R} \) is a family of solutions, then \( (x, \lambda) \in T \) is a bifurcation point with respect to \( T \) if there is a sequence of solutions \( (x_n, \lambda_n) \notin T \) converging to \( (x, \lambda) \). In this case one speaks of (primary) bifurcation with respect to \( T \) and there are many powerful theorems that allow to detect such bifurcations under suitable assumptions on \( F \). Examples for such theorems are the celebrated bifurcation results due to

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