

**EXISTENCE OF POSITIVE SOLUTIONS  
FOR HARDY NONLOCAL FRACTIONAL  
ELLIPTIC EQUATIONS  
INVOLVING CRITICAL NONLINEARITIES**

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ABSTRACT. In this paper, we have used variational methods to study existence of solutions for the following critical nonlocal fractional Hardy elliptic equation

$$(-\Delta)^s u - \gamma \frac{u}{|x|^{2s}} = \frac{|u|^{2_s^*(b)-2} u}{|x|^b} + \lambda f(x, u), \quad \text{in } \mathbb{R}^N,$$

where  $N > 2s$ ,  $0 < s < 1$ ,  $\gamma, \lambda$  are real parameters,  $(-\Delta)^s$  is the fractional Laplace operator,  $2_s^*(b) = 2(N-b)/(N-2s)$  is a critical Hardy–Sobolev exponent with  $b \in [0, 2s)$  and  $f \in C(\mathbb{R}^N \times \mathbb{R}, \mathbb{R})$ .

## 1. Introduction

This paper is devoted to the study of the following critical nonlocal fractional Hardy elliptic equation

$$(1.1) \quad (-\Delta)^s u - \gamma \frac{u}{|x|^{2s}} = \frac{|u|^{2_s^*(b)-2} u}{|x|^b} + \lambda f(x, u), \quad \text{in } \mathbb{R}^N,$$

where  $N > 2s$ ,  $0 < s < 1$ ,  $\gamma, \lambda$  are real parameters,  $2_s^*(b) = 2(N-b)/(N-2s)$  is a critical Hardy–Sobolev exponent with  $b \in [0, 2s)$ ,  $2_s^* = 2_s^*(0) = 2N/(N-2s)$

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