# EXISTENCE OF POSITIVE SOLUTIONS
# FOR HARDY NONLOCAL FRACTIONAL
# ELLIPTIC EQUATIONS
# INVOLVING CRITICAL NONLINEARITIES

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Abstract. In this paper, we have used variational methods to study existence of solutions for the following critical nonlocal fractional Hardy elliptic equation

$$(-\Delta)^s u - \gamma \frac{u}{|x|^{2s}} = \frac{|u|^{2^*_s(b)-2}u}{|x|^b} + \lambda f(x,u), \quad \text{in } \mathbb{R}^N,$$

where $N > 2s$, $0 < s < 1$, $\gamma, \lambda$ are real parameters, $(-\Delta)^s$ is the fractional Laplace operator, $2^*_s(b) = 2(N-b)/(N-2s)$ is a critical Hardy–Sobolev exponent with $b \in [0, 2s)$ and $f \in C(\mathbb{R}^N \times \mathbb{R}, \mathbb{R})$.

## 1. Introduction

This paper is devoted to the study of the following critical nonlocal fractional Hardy elliptic equation

$$(1.1) \qquad (-\Delta)^s u - \gamma \frac{u}{|x|^{2s}} = \frac{|u|^{2^*_s(b)-2}u}{|x|^b} + \lambda f(x,u), \quad \text{in } \mathbb{R}^N,$$

where $N > 2s$, $0 < s < 1$, $\gamma, \lambda$ are real parameters, $2^*_s(b) = 2(N-b)/(N-2s)$ is a critical Hardy–Sobolev exponent with $b \in [0, 2s)$, $2^*_s = 2^*_s(0) = 2N/(N-2s)$