Topological **M**ethods in **N**onlinear **A**nalysis
Volume 53, No. 2, 2019, 697–729
DOI: 10.12775/TMNA.2019.018

ON FINDING THE GROUND STATE SOLUTION TO THE LINEARLY COUPLED BREZIS–NIRENBERG SYSTEM IN HIGH DIMENSIONS: THE COOPERATIVE CASE

YUANZE WU

ABSTRACT. Consider the following elliptic system

$$\begin{cases} -\Delta u_i + \mu_i u_i = |u_i|^{2^*-2}u_i + \lambda \sum\limits_{j=1,\, j\neq i}^{k} u_j & \text{in } \Omega, \\ u_i = 0, \quad i = 1, \ldots, k, & \text{on } \partial\Omega, \end{cases}$$

where $k \geq 2$, $\Omega \subset \mathbb{R}^N$ $(N \geq 4)$ is a bounded domain with smooth boundary $\partial\Omega$, $2^* = 2N/(N-2)$ is the Sobolev critical exponent, $\mu_i \in \mathbb{R}$ for all $i = 1, \ldots, k$ are constants and $\lambda \in \mathbb{R}$ is a parameter. By the variational method, we mainly prove that the above system has a ground state for all $\lambda > 0$. Our results reveal some new properties of the above system that imply that the parameter λ plays the same role as in the following well-known Brezis–Nirenberg equation

$$\begin{cases} -\Delta u = \lambda u + |u|^{2^*-2}u & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases}$$

and this system has a very similar structure of solutions as the above Brezis–Nirenberg equation for λ.

2010 *Mathematics Subject Classification.* 35B09; 35B33; 35J50.
Key words and phrases. Elliptic system; linearly coupled; ground state; variational method; Sobolev critical exponent.