POSITIVE LEAST ENERGY SOLUTIONS
FOR COUPLED NONLINEAR CHOQUARD EQUATIONS
WITH HARDY–LITTLEWOOD–SOBOLEV CRITICAL EXPONENT

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Abstract. In this paper, we study the existence and nonexistence of positive least energy solutions of the following coupled nonlinear Schrödinger equations with Choquard type nonlinearities:

\begin{align*}
-\Delta u + \nu_1 u &= \mu_1 \left( \frac{1}{|x|^k} \ast u^2 \right) u + \beta \left( \frac{1}{|x|^k} \ast v^2 \right) u, \quad x \in \Omega, \\
-\Delta v + \nu_2 v &= \mu_2 \left( \frac{1}{|x|^k} \ast u^2 \right) v + \beta \left( \frac{1}{|x|^k} \ast u^2 \right) v, \quad x \in \Omega, \\
u, v &\geq 0 \quad \text{in } \Omega, \\
u = v = 0 \quad \text{on } \partial \Omega.
\end{align*}

Here \( \Omega \subset \mathbb{R}^N \) is a smooth bounded domain, \(-\lambda_1(\Omega) < \nu_1, \nu_2 < 0, \lambda_1(\Omega)\) is the first eigenvalue of \((-\Delta, H^k_0(\Omega))\), \(\mu_1, \mu_2 > 0\) and \(\beta \neq 0\) is a coupling constant. We show that the critical nonlocal elliptic system has a positive least energy solution under appropriate conditions on parameters via variational methods. For the case in which \(\nu_1 = \nu_2\), we obtain the classification of the positive least energy solutions. Moreover, the asymptotic behaviors of the positive least energy solutions as \(\beta \to 0\) are studied.

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623