Topological Methods in Nonlinear Analysis
Volume 53, No. 2, 2019, 519–545
DOI: 10.12775/TMNA.2019.010

ON GROUND STATE SOLUTIONS FOR THE NONLINEAR KIRCHHOFF TYPE PROBLEMS WITH A GENERAL CRITICAL NONLINEARITY

Weihong Xie — Haibo Chen

Abstract. In this paper, we are concerned with the following Kirchhoff type problem with critical growth:

$$-\left(a+b\int_{\mathbb{R}^3}|\nabla u|^2\,dx\right)\Delta u+V(x)u=f(u)+|u|^4u,\quad u\in H^1(\mathbb{R}^3),$$

where $a,b>0$ are constants. Under certain assumptions on V and f, we prove that the above problem has a ground state solution of Nehari–Pohozaev type and a least energy solution via variational methods. Furthermore, we also show that the mountain pass value gives the least energy level for the above problem. Our results improve and extend some recent ones in the literature.

1. Introduction and statement of results

In this paper, we study the existence of ground state solutions for the following Kirchhoff type problem with a critical nonlinearity:

$$-\left(a+b\int_{\mathbb{R}^3}|\nabla u|^2\,dx\right)\Delta u+V(x)u=f(u)+|u|^4u,\quad x\in\mathbb{R}^3. \tag{1.1}$$

2010 *Mathematics Subject Classification.* 35J20, 35J65.

Key words and phrases. Kirchhoff type problems; ground state solutions of Nehari–Pohozaev type; the least energy solutions; variational methods; critical Sobolev exponent.

H. Chen is supported by the National Natural Science Foundation of China (No. 11671403).

W. Xie is supported by the Fundamental Research Funds for the Central Universities of Central South University (No. 2018zzts006).