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NONAUTONOMOUS CONLEY INDEX THEORY THE CONNECTING HOMOMORPHISM

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Abstract. Attractor-repeller decompositions of isolated invariant sets give rise to so-called connecting homomorphisms. These homomorphisms reveal information on the existence and strucuture of connecting trajectories of the underlying dynamical system.
To give a meaningful generalization of this general principle to nonautonomous problems, the nonautonomous homology Conley index is expressed as a direct limit. Moreover, it is shown that a nontrivial connecting homomorphism implies, on the dynamical systems level, a sort of uniform connectedness of the attractor-repeller decomposition.

In previous works [6], [7] the author developed a nonautonomous Conley index theory. The index relies on the interplay between a skew-product semiflow and a nonautonomous evolution operator. It can be applied to various nonautonomous problems, including ordinary differential equations and semilinear parabolic equations (see [6]).

Every attractor-repeller decomposition of an isolated invariant set gives rise to a long exact sequence involving the homology Conley index. The connecting homomorphism of this sequence contains information on the connections between

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