ON POSITIVE VISCOSITY SOLUTIONS
OF FRACTIONAL LANE–EMDEN SYSTEMS

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ABSTRACT. In this paper we discuss the existence, nonexistence and uniqueness of positive viscosity solution for the following coupled system involving fractional Laplace operator on a smooth bounded domain $\Omega$ in $\mathbb{R}^n$:

$$
\begin{align*}
(-\Delta)^s u - \Delta^p u &= 0 & & \text{in } \Omega, \\
(-\Delta)^s v - \Delta^q v &= 0 & & \text{in } \Omega, \\
u = v = 0 & & \text{in } \mathbb{R}^n \setminus \Omega.
\end{align*}
$$

By means of an appropriate variational framework and a Hölder regularity result for suitable weak solutions of the above system, we prove that such a system admits at least one positive viscosity solution for any $0 < s < 1$, provided that $p, q > 0$, $pq \neq 1$ and the couple $(p, q)$ is below the critical hyperbole

$$
\frac{1}{p+1} + \frac{1}{q+1} = \frac{n - 2s}{n}
$$

whenever $n > 2s$. Moreover, by using the maximum principles for the fractional Laplace operator, we show that uniqueness occurs whenever $pq < 1$. Lastly, assuming $\Omega$ is star-shaped, by using a Rellich type variational identity, we prove that no such a solution exists if $(p, q)$ is on or above the critical hyperbole. A crucial point in our proofs is proving, given a critical point $u \in W^{s, (p+1)/p}(\Omega) \cap W^{2s, (p+1)/p}(\Omega)$ of a related functional, that there is a function $v$ in an appropriate Sobolev space (Proposition 2.1) so that $(u, v)$ is a weak solution of the above system and a bootstrap argument can be applied successfully in order to establish its Hölder regularity (Proposition 3.1). The difficulty is caused mainly by the absence of a $L^p$ Calderón–Zygmund theory with $p > 1$ associated to the operator $(-\Delta)^s$ for $0 < s < 1$.

2010 Mathematics Subject Classification. Primary: 35J90, 35J60; Secondary: 49K20.

Key words and phrases. Fractional problems; critical hyperbole; Lane–Emden system; existence; uniqueness.

The first author was partially supported by Fapemig. The second author was partially supported by CNPq (PQ 306855/2016-0) and Fapemig (APQ 02574-16).