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ON POSITIVE VISCOSITY SOLUTIONS OF FRACTIONAL LANE–EMDEN SYSTEMS

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Abstract. In this paper we discuss the existence, nonexistence and uniqueness of positive viscosity solution for the following coupled system involving fractional Laplace operator on a smooth bounded domain Ω in $\mathbb{R}^n$:

$$\begin{cases} (-\Delta)^s u = v^p & \text{in } \Omega, \\ (-\Delta)^s v = u^q & \text{in } \Omega, \\ u = v = 0 & \text{in } \mathbb{R}^n \setminus \Omega. \end{cases}$$

By means of an appropriate variational framework and a Hölder regularity result for suitable weak solutions of the above system, we prove that such a system admits at least one positive viscosity solution for any $0 < s < 1$, provided that $p, q > 0$, $pq \neq 1$ and the couple (p, q) is below the critical hyperbole

$$\frac{1}{p+1} + \frac{1}{q+1} = \frac{n-2s}{n}$$

whenever $n > 2s$. Moreover, by using the maximum principles for the fractional Laplace operator, we show that uniqueness occurs whenever $pq < 1$. Lastly, assuming Ω is star-shaped, by using a Rellich type variational identity, we prove that no such a solution exists if (p, q) is on or above the critical hyperbole. A crucial point in our proofs is proving, given a critical point $u \in W_0^{s,(p+1)/p}(\Omega) \cap W^{2s,(p+1)/p}(\Omega)$ of a related functional, that there is a function v in an appropriate Sobolev space (Proposition 2.1) so that (u, v) is a weak solution of the above system and a bootstrap argument can be applied successfully in order to establish its Hölder regularity (Proposition 3.1). The difficulty is caused mainly by the absence of a L^p Calderón–Zygmund theory with $p > 1$ associated to the operator $(-\Delta)^s$ for $0 < s < 1$.

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