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ZERO-DIMENSIONAL COMPACT METRIZABLE SPACES AS ATTRACTORS OF GENERALIZED ITERATED FUNCTION SYSTEMS

Filip Strobin — Łukasz Maślanka

Abstract. R. Miculescu and A. Mihail in 2008 introduced the concept of a *generalized iterated function system* (GIFS in short), a particular extension of the classical IFS. The idea is that, instead of families of selfmaps of a metric space X, GIFSs consist of maps defined on a finite Cartesian m-th power X^m with values in X (in such a case we say that a GIFS is *of order* m). It turned out that a great part of the classical Hutchinson theory has natural counterpart in this GIFSs' framework. On the other hand, there are known only few examples of fractal sets which are generated by GIFSs, but which are not IFSs' attractors. In the paper we study 0-dimensional compact metrizable spaces from the perspective of GIFSs' theory. Such investigations for classical IFSs have been undertaken in the last several years, for example by T. Banakh, E. D'Aniello, M. Nowak, T.H. Steele and F. Strobin.

We prove that each such space X is homeomorphic to the attractor of some GIFS on the real line. Moreover, we prove that X can be embedded into the real line $\mathbb{R}$ as the attractor of some GIFS of order m and (in the same time) a nonattractor of any GIFS of order $m-1$, as well as it can be embedded as a nonattractor of any GIFS. Then we show that a relatively simple modifications of X deliver spaces whose each connected component is "big" and which are GIFS's attractors not homeomorphic with IFS's attractors. Finally, we use obtained results to show that a generic compact subset of a Hilbert space is not the attractor of any Banach GIFS.

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