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A DIFFUSIVE LOGISTIC EQUATION WITH U-SHAPED DENSITY DEPENDENT DISPERSAL ON THE BOUNDARY

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Abstract. We study positive solutions to the steady state reaction diffusion equation:

$$\begin{cases} -\Delta v = \lambda v(1-v), & x \in \Omega_0, \\ \dfrac{\partial v}{\partial \eta} + \gamma\sqrt{\lambda}(v-A)^2 v = 0, & x \in \partial\Omega_0, \end{cases}$$

where Ω_0 is a bounded domain in $\mathbb{R}^n$; $n \geq 1$ with smooth boundary $\partial\Omega_0$, $\partial/\partial\eta$ is the outward normal derivative, $A \in (0,1)$ is a constant, and λ, γ are positive parameters. Such models arise in the study of population dynamics when the population exhibits a U-shaped density dependent dispersal on the boundary of the habitat. We establish existence, multiplicity, and uniqueness results for certain ranges of the parameters λ and γ. We obtain our existence and mulitplicity results via the method of sub-super solutions.

1. Introduction

Let $\Omega_0 = (0,1)$ or be a bounded domain in $\mathbb{R}^n$; $n = 2, 3$ with smooth boundary $\partial\Omega_0$ and $|\Omega_0| = 1$. Let $\Omega = \{\ell x \mid x \in \Omega_0\}$, where ℓ is a positive parameter representing the patch size of Ω. We will consider a population that satisfies

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