A DIFFUSIVE LOGISTIC EQUATION  
WITH U-SHAPED DENSITY DEPENDENT DISPERSAL  
ON THE BOUNDARY

JEROME GODDARD II — QUINN MORRIS  
CATHERINE PAYNE — RATNASINGHAM SHIVAJI

**Abstract.** We study positive solutions to the steady state reaction diffusion equation:

\[
\begin{aligned}
-\Delta v &= \lambda v(1 - v), & x &\in \Omega_0, \\
\frac{\partial v}{\partial \eta} + \gamma \sqrt{\lambda(v - A)}^2 v &= 0, & x &\in \partial \Omega_0,
\end{aligned}
\]

where \( \Omega_0 \) is a bounded domain in \( \mathbb{R}^n; n \geq 1 \) with smooth boundary \( \partial \Omega_0 \), \( \partial / \partial \eta \) is the outward normal derivative, \( A \in (0, 1) \) is a constant, and \( \lambda, \gamma \) are positive parameters. Such models arise in the study of population dynamics when the population exhibits a U-shaped density dependent dispersal on the boundary of the habitat. We establish existence, multiplicity, and uniqueness results for certain ranges of the parameters \( \lambda \) and \( \gamma \). We obtain our existence and multiplicity results via the method of sub-super solutions.

1. Introduction

Let \( \Omega_0 = (0, 1) \) or be a bounded domain in \( \mathbb{R}^n; n = 2, 3 \) with smooth boundary \( \partial \Omega_0 \) and \( |\Omega_0| = 1 \). Let \( \Omega = \{ \ell x \mid x \in \Omega_0 \} \), where \( \ell \) is a positive parameter representing the patch size of \( \Omega \). We will consider a population that satisfies

2010 *Mathematics Subject Classification.* 35J25, 35J66.

**Key words and phrases.** Mathematical biology; reaction diffusion model; nonlinear boundary conditions; U-shaped density dependent dispersal.

This material is based upon work supported by the National Science Foundation under Grants No. DMS-1516519 and DMS-1516560.