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A GENERIC RESULT ON WEYL TENSOR

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Abstract. Let M be a connected compact C^∞ manifold of dimension $n \geq 4$ without boundary. Let $\mathcal{M}^k$ be the set of all C^k Riemannian metrics on M. Any $g \in \mathcal{M}^k$ determines the Weyl tensor

$$\mathcal{W}^g \colon M \to \mathbb{R}^{4n}, \qquad \mathcal{W}^g(\xi) := (W^g_{ijkl}(\xi))_{i,j,k,l=1,\ldots,n}.$$

We prove that the set

$$\mathcal{A} := \big\{g \in \mathcal{M}^k : |\mathcal{W}^g(\xi)| + |D\mathcal{W}^g(\xi)| + |D^2\mathcal{W}^g(\xi)| > 0 \text{ for any } \xi \in M\big\}$$

is an open dense subset of $\mathcal{M}^k$.

1. Introduction

Let M be a connected compact C^∞ manifold of dimension $n \geq 4$ without boundary. Let $\mathcal{M}^k$ be the set of all C^k Riemannian metrics on M. Any $g \in \mathcal{M}^k$ determines the Weyl tensor

$$\mathcal{W}^g \colon M \to \mathbb{R}^{4n}, \qquad \mathcal{W}^g(\xi) := (W^g_{ijkl}(\xi))_{i,j,k,l=1,\ldots,n}.$$

Our goal is to prove that, for a generic Riemannian metric g, it holds true that if Weyl tensor and its first derivative vanish at a point $\xi \in M$ then the second derivative at ξ is not zero. More precisely, we prove that

Theorem 1.1. *The set*

$$\mathcal{A} := \Big\{g \in \mathcal{M}^k : \min_{\xi \in M}(|\mathcal{W}^g(\xi)| + |D\mathcal{W}^g(\xi)| + |D^2\mathcal{W}^g(\xi)|) > 0\Big\}$$

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