Topological Methods in Nonlinear Analysis
Volume 53, No. 1, 2019, 151–182
DOI: 10.12775/TMNA.2018.043

MULTIPLICITY OF POSITIVE SOLUTIONS FOR FRACTIONAL LAPLACIAN EQUATIONS INVOLVING CRITICAL NONLINEARITY

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Abstract. In this paper, we consider the following problem involving fractional Laplacian operator

$$(-\Delta)^s u = \lambda f(x)|u|^{q-2}u + |u|^{2_s^*-2}u \quad \text{in } \Omega, \qquad u = 0 \quad \text{on } \partial\Omega,$$

where Ω is a smooth bounded domain in $\mathbb{R}^N$, $0 < s < 1$, $2_s^* = 2N/(N-2s)$, and $(-\Delta)^s$ is the fractional Laplacian. We will prove that there exists $\lambda_* > 0$ such that the problem has at least two positive solutions for each $\lambda \in (0, \lambda_*)$. In addition, the concentration behavior of the solutions are investigated.

1. Introduction

In this paper, we consider the following problem with the fractional Laplacian:

$$(1.1)\qquad \begin{cases} (-\Delta)^s u = \lambda f(x)|u|^{q-2}u + |u|^{2_s^*-2}u & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases}$$

where Ω is a smooth bounded domain in $\mathbb{R}^N$, $N > 2s$, $0 < s < 1$, $1 < q < 2$, $\lambda > 0$, $2_s^* := 2N/(N-2s)$ is the critical exponent in fractional Sobolev inequalities, and $f\colon \Omega \to \mathbb{R}$ is a continuous function with $f^+(x) = \max\{f(x), 0\} \neq 0$ on Ω, and $f \in L^{2_s^*/(2_s^*-q)}(\Omega)$. From the assumptions on f and q, we know that

2010 *Mathematics Subject Classification.* 35J60; 47J30.
Key words and phrases. Fractional Laplacian equation; critical Sobolev exponent; variational methods.