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OPTIMAL RETRACTION PROBLEM FOR PROPER k-BALL-CONTRACTIVE MAPPINGS IN $C^m[0,1]$

Diana Caponetti — Alessandro Trombetta — Giulio Trombetta

Abstract. In this paper for any $\varepsilon > 0$ we construct a new proper k-ball-contractive retraction of the closed unit ball of the Banach space $C^m[0,1]$ onto its boundary with $k < 1+\varepsilon$, so that the Wośko constant $W_\gamma(C^m[0,1])$ is equal to 1.

1. Introduction and preliminaries

Let X be an infinite-dimensional Banach space with the closed unit ball $B(X)$ and the unit sphere $S(X)$. After two works by Klee [22] and [23] it is known that there exists a *retraction* $R\colon B(X) \to S(X)$, i.e. R is a continuous mapping such that $Rx = x$, for all $x \in S(X)$. As concerns the metric properties of such retractions Benyamini and Sternfeld ([5]), following Nowak ([24]), have obtained the remarkable result that for every Banach space X there exists a retraction of $B(X)$ onto $S(X)$ satisfying, for some constant L, the L-Lipschitz condition

$$\|Rx - Ry\| \le L\|x - y\| \quad \text{for all } x, y \in B(X).$$

Clearly the same is not true for spaces of finite dimension due to the Brouwer's Non Retraction Theorem. The optimal retraction problem, considered for the first time in [17], consists in the evaluation of the constant

$$k_0(X) = \inf\{L : \text{there is a } L\text{-Lipschitz retraction } R\colon B(X) \to S(X)\}.$$

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