Topological Methods in Nonlinear Analysis
Volume 53, No. 1, 2019, 79–96
DOI: 10.12775/TMNA.2018.040

NONAUTONOMOUS CONLEY INDEX THEORY. CONTINUATION OF MORSE-DECOMPOSITIONS

Axel Jänig

Abstract. In previous works the author established a nonautonomous Conley index based on the interplay between a nonautonomous evolution operator and its skew-product formulation. In this paper, the treatment of attractor–repeller decomposition is refined. The more general concept of partially ordered Morse-decompositions is used. It is shown that, in the nonautonomous setting, these Morse-decompositions persist under small perturbations. Furthermore, a continuation property for these Morse decompositions is established. Roughly speaking, the index of every Morse-set and every connecting homomorphism continue as the nonautonomous problem, depending continuously on a parameter, changes.

In previous works [5], [6] the author developed a nonautonomous Conley index theory. The index relies on the interplay between a skew-product semiflow and a nonautonomous evolution operator. It can be applied to various nonautonomous problems, including ordinary differential equations and semilinear parabolic equations (see [5]).

There are multiple variants such as a homotopy index, a homology Conley index or a categorial index. In [6], also attractor–repeller decompositions of isolated invariant sets are introduced. In particular, every attractor–repeller decomposition of an isolated invariant set gives rise to a long exact sequence

2010 *Mathematics Subject Classification.* Primary: 37B30, 37B55; Secondary: 34C99, 35B08, 35B40, 35B99.

Key words and phrases. Nonautonomous differential equations; attractor–repeller decompositions; Morse–Conley index theory; partially orderd Morse-decompositions; homology index braid; continuation property; nonautonomous Conley index; homology Conley index.