FINITE-TIME BLOW-UP
IN A QUASILINEAR CHEMOTAXIS SYSTEM
WITH AN EXTERNAL SIGNAL CONSUMPTION

PAN ZHENG — CHUNLAI MU — XUEGANG HU — LIANGCHEN WANG

ABSTRACT. This paper deals with a quasilinear chemotaxis system with an external signal consumption

\[ \begin{cases}
  u_t = \nabla \cdot (\varphi(u) \nabla u) - \nabla \cdot (u \nabla v), & (x, t) \in \Omega \times (0, \infty), \\
  \theta = \Delta v + u - g(x), & (x, t) \in \Omega \times (0, \infty),
\end{cases} \]

under homogeneous Neumann boundary conditions in a ball \( \Omega \subset \mathbb{R}^n \), where \( \varphi(u) \) is a nonlinear diffusion function and \( g(x) \) is an external signal consumption. Under suitable assumptions on the functions \( \varphi \) and \( g \), it is proved that there exists initial data such that the solution of the above system blows up in finite time.

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