Topological **M**ethods in **N**onlinear **A**nalysis
Volume 53, No. 1, 2019, 25–41
DOI: 10.12775/TMNA.2018.035

FINITE-TIME BLOW-UP IN A QUASILINEAR CHEMOTAXIS SYSTEM WITH AN EXTERNAL SIGNAL CONSUMPTION

Pan Zheng — Chunlai Mu — Xuegang Hu — Liangchen Wang

Abstract. This paper deals with a quasilinear chemotaxis system with an external signal consumption

$$\begin{cases} u_t = \nabla \cdot (\varphi(u)\nabla u) - \nabla \cdot (u\nabla v), & (x,t) \in \Omega \times (0,\infty), \\ 0 = \Delta v + u - g(x), & (x,t) \in \Omega \times (0,\infty), \end{cases}$$

under homogeneous Neumann boundary conditions in a ball $\Omega \subset \mathbb{R}^n$, where $\varphi(u)$ is a nonlinear diffusion function and $g(x)$ is an external signal consumption. Under suitable assumptions on the functions φ and g, it is proved that there exists initial data such that the solution of the above system blows up in finite time.

2010 *Mathematics Subject Classification.* 35K40, 35K55, 35B35, 35B40, 92C17.

Key words and phrases. Finite-time blow-up; chemotaxis; external signal consumption.

The first author is partially supported by National Natural Science Foundation of China (Grant Nos: 11601053, 11526042), the Scientific and Technological Research Program of Chongqing Municipal Education Commission (Grant No: KJ1500403), the Basic and Advanced Research Project of CQCSTC (Grant No: cstc2015jcyjA00008), and the Doctor Start-up Funding and the Natural Science Foundation of Chongqing University of Posts and Telecommunications (Grant Nos: A2014-25 and A2014-106).

The second author is partially supported by National Natural Science Foundation of China (Grant Nos.: 11771062 , 11571062), the Basic and Advanced Research Project of CQCSTC (Grant No: cstc2015jcyjBX0007) and the Fundamental Research Funds for the Central Universities (Grant No: 10611CDJXZ238826).

The third author is partially supported by the Basic and Advanced Research Project of CQCSTC (Grant No: cstc2017jcyjBX0037). The fourth author is partially supported by National Natural Science Foundation of China (Grant Nos: 11601052).