HETEROCLINIC SOLUTIONS
OF ALLEN–CAHN TYPE EQUATIONS
WITH A GENERAL ELLIPTIC OPERATOR

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ABSTRACT. We consider a generalization of the Allen–Cahn type equation in
divergence form $-\text{div}(\nabla G(\nabla u(x,y))) + F_u(x,y,u(x,y)) = 0$. This is
more general than the usual Laplace operator. We prove the existence and
regularity of heteroclinic solutions under standard ellipticity and $m$-growth
conditions.

1. Introduction

The Allen–Cahn equation is a well-known elliptic partial differential equation
considered by many authors in the form:

$$-\Delta u(x,y) + F_u(x,y,u) = 0$$

where $F$ is a double-well potential of $u$ and has some other standard properties
like periodicity in $x$ and $y$ (see the next section for details). Here we are not
interested in the Dirichlet problem but in the existence of heteroclinic solutions
in the whole of $\mathbb{R}^2$. This problem was widely studied and there are many articles
that contain the existence theorems about such solutions. As an example we can
take [11] where the authors show the existence and multiplicity of heteroclinic
and some other special types of solutions. Earlier in [1] and [2] the problem was
solved in a more simple form where $F'(x,u) = f(x)F'(u)$.

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