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A CONTINUATION LEMMA AND THE EXISTENCE OF PERIODIC SOLUTIONS OF PERTURBED PLANAR HAMILTONIAN SYSTEMS WITH SUB-QUADRATIC POTENTIALS

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Abstract. In this paper, we study the existence of periodic solutions of perturbed planar Hamiltonian systems of the form

$$\begin{cases} x' = f(y) + p_1(t, x, y), \\ y' = -g(x) + p_2(t, x, y). \end{cases}$$

We prove a continuation lemma for a given planar system and further use it to prove that this system has at least one T-periodic solution provided that g has some sub-quadratic potentials.

1. Introduction

We are concerned with the existence of periodic solutions of a perturbed planar Hamiltonian system

$$\begin{cases} x' = f(y) + p_1(t, x, y), \\ y' = -g(x) + p_2(t, x, y), \end{cases} \tag{1.1}$$

where $f, g\colon \mathbb{R} \to \mathbb{R}$ are continuous, $p_i\colon \mathbb{R}^3 \to \mathbb{R}$ $(i = 1, 2)$ are continuous and T-periodic with respect to the first variable.

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