NONZERO POSITIVE SOLUTIONS
OF A MULTI-PARAMETER ELLIPTIC SYSTEM
WITH FUNCTIONAL BCS

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This paper is dedicated to the memory of the late Panagiotis K. Palamides,
for his teachings and friendship

ABSTRACT. We prove, by topological methods, new results on the existence
of nonzero positive weak solutions for a class of multi-parameter second or-
der elliptic systems subject to functional boundary conditions. The setting
is fairly general and covers the case of multi-point, integral and nonlinear
boundary conditions. We also present a non-existence result. We provide
some examples to illustrate the applicability of our theoretical results.

1. Introduction

In this paper we discuss the solvability of the multi-parameter system of
second order elliptic equations subject to functional boundary conditions (BCs)

\[ \begin{aligned}
L_i u_i(x) &= \lambda_i f_i(x, u(x)), & x \in \Omega, \quad i = 1, \ldots, n, \\
B_i u_i(x) &= \eta_i h_i[u], & x \in \partial \Omega, \quad i = 1, \ldots, n,
\end{aligned} \tag{1.1} \]

where $\Omega \subset \mathbb{R}^m$ ($m \geq 2$) is a bounded domain with sufficiently regular boundary,
$L_i$ is a strongly uniformly elliptic operator, $B_i$ is a first order boundary operator,

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