NONZERO POSITIVE SOLUTIONS
OF A MULTI-PARAMETER ELLIPTIC SYSTEM
WITH FUNCTIONAL BCS

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*This paper is dedicated to the memory of the late Panagiotis K. Palamides,
for his teachings and friendship*

ABSTRACT. We prove, by topological methods, new results on the existence
of nonzero positive weak solutions for a class of multi-parameter second or-
der elliptic systems subject to functional boundary conditions. The setting
is fairly general and covers the case of multi-point, integral and nonlinear
boundary conditions. We also present a non-existence result. We provide
some examples to illustrate the applicability of our theoretical results.

1. Introduction

In this paper we discuss the solvability of the multi-parameter system of
second order elliptic equations subject to functional boundary conditions (BCs)

$$(1.1) \quad \begin{cases} L_i u_i(x) = \lambda_i f_i(x, u(x)), & x \in \Omega, \quad i = 1, \ldots, n, \\ B_i u_i(x) = \eta_i h_i[u], & x \in \partial\Omega, \quad i = 1, \ldots, n, \end{cases}$$

where $\Omega \subset \mathbb{R}^m$ ($m \geq 2$) is a bounded domain with sufficiently regular boundary,
L_i is a strongly uniformly elliptic operator, B_i is a first order boundary operator,