Topological **M**ethods in **N**onlinear **A**nalysis
Volume 52, No. 2, 2018, 631–664
DOI: 10.12775/TMNA.2018.025

ON SPECTRAL CONVERGENCE FOR SOME PARABOLIC PROBLEMS WITH LOCALLY LARGE DIFFUSION

Maria C. Carbinatto — Krzysztof P. Rybakowski

Abstract. In this paper, which is a sequel to [1], we extend the spectral convergence result from [5] to a larger class of singularly perturbed families of scalar linear differential operators. This also extends the Conley index continuation principles from [1].

1. Introduction

In the important paper [5], Carvalho and Pereira approached a problem previously considered by Fusco [6] from the point of view of spectral convergence. Specifically, they considered a family of linear differential operators $u \mapsto -(a_\varepsilon u_x)_x$ on the interval $]0,1[$ with boundary conditions

$$(\mathrm{S}_\varepsilon) \qquad \begin{cases} \rho u - (1-\rho) a_\varepsilon u_x = 0, & x = 0, \\ \sigma u + (1-\sigma) a_\varepsilon u_x = 0, & x = 1, \end{cases}$$

and made the following

Assumption 1.1. $n \in \mathbb{N}$, $\varepsilon_0 \in\,]0,\infty]$, $(e_j)_{j\in[1..n]}$, $(l_j)_{j\in[0..n]}$, $(b_j)_{j\in[0..n]}$ are sequences of positive constants and $(l'_j)_{j\in[0..n]}$, $(b'_j)_{j\in[0..n]}$ are sequences of positive functions defined on $]0,\varepsilon_0[$ such that $l'_j(\varepsilon) > l_j$ and $b'_j(\varepsilon) > b_j$ for $j \in$

2010 *Mathematics Subject Classification.* Primary: 35P99, 35K57; Secondary: 37B30, 35B25.
Key words and phrases. Spectral convergence; localized large diffusion; singular perturbations; Conley index.