CONVENIENT MAPS
FROM ONE-RELATOR MODEL TWO-COMPLEXES
INTO THE REAL PROJECTIVE PLANE

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ABSTRACT. Let f be a map from a one-relator model two-complex $K_\mathcal{P}$ into the real projective plane. The composition $\varrho \circ f_\#$ of the homomorphism $f_\#$ induced by f on fundamental groups with the action ϱ of $\pi_1(\mathbb{R}\mathrm{P}^2)$ over $\pi_2(\mathbb{R}\mathrm{P}^2)$ provides a local integer coefficient system $f_\#^\varrho$ over $K_\mathcal{P}$. We prove that if the twisted integer cohomology group $H^2(K_\mathcal{P}; {}_{f_\#^\varrho}\mathbb{Z}) = 0$, then f is homotopic to a non-surjective map. As an intermediary step for the proof, we show that if $H^2(K_\mathcal{P}; {}_\beta\mathbb{Z}) = 0$ for some local integer coefficient system β over $K_\mathcal{P}$, then $K_\mathcal{P}$ is aspherical.

1. Introduction

The existence of strong surjections from a finite and connected n-dimensional CW complex K (a n-complex, to shorten) into a closed n-manifold Y has been investigated for at least a decade, specially from the viewpoint of the topological root theory.

For a *strong surjection* from K into Y we mean a (continuous) map $f\colon K \to Y$ whose free homotopy class $[f] \in [K; Y]$ has just surjective maps. In this case, we say also that f is *strongly surjective*. In the context of topological root theory, a map $f\colon K \to Y$ which is not strongly surjective is said to be *root free*.