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CONTRACTIBILITY OF MANIFOLDS BY MEANS OF STOCHASTIC FLOWS

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Abstract. In the paper [Probab. Theory Relat. Fields, **100** (1994), 417–428] Xue-Mei Li has shown that the moment stability of an SDE is closely connected with the topology of the underlying manifold. In particular, she gave sufficient condition on SDE on a manifold M under which the fundamental group $\pi_1 M = 0$. We prove that under similar analytical conditions the manifold M is contractible, that is all homotopy groups $\pi_n M$, $n \geq 1$, vanish.

1. Introduction

The interplay between geometrical or topological structures of a manifold and the properties of differential operations on it forms a library of the most crucial results in analysis. For instance,

(1) if M is closed, then the number of (non-degenerate) critical points of index i of a Morse function $f\colon M \to \mathbb{R}$ bounds the rank of i-th homology group of M (Morse inequalities);
(2) de Rham cohomologies $H^*_{\mathrm{DR}}(M)$ of an orientable manifold M are isomorphic with its singular real cohomologies $H^*(M,\mathbb{R})$, (de Rham theory);

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