Topological **M**ethods in **N**onlinear **A**nalysis
Volume 52, No. 2, 2018, 515–540
DOI: 10.12775/TMNA.2018.014

POINTWISE ESTIMATES IN THE FILIPPOV LEMMA AND FILIPPOV–WAŻEWSKI THEOREM FOR FOURTH ORDER DIFFERENTIAL INCLUSIONS

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Abstract. In this work we give a generalization of the Filippov–Ważewski Theorem to the fourth order differential inclusions in a separable complex Banach space $\mathbb{X}$

$$\mathcal{D}y = y'''' - (A^2 + B^2)y'' + A^2B^2y \in F(t, y),$$

with the initial conditions in $c \in [0, T]$

$$y(c) = \alpha, \qquad y'(c) = \beta, \qquad y''(c) = \gamma, \qquad y'''(c) = \delta, \tag{0.1}$$

We assume that the multifunction $F : [0, T] \times \mathbb{X} \rightsquigarrow c(\mathbb{X})$ is Lipschitz continuous in y with the integrable Lipschitz constant $l(\,\cdot\,)$, while $A^2, B^2 \in B(\mathbb{X})$ are the infinitesimal generators of two cosine families of operators. The main result is the following version of Filippov Lemma:

Theorem. *Let $y_0 \in W^{4,1} = W^{4,1}([0, T], \mathbb{X})$ be such function with* (0.1) *that*

$$\operatorname{dist}(\mathcal{D}y_0(t), F(t, y_0(t))) \le p_0(t) \quad \textit{a.e. in } [c, d] \subset [0, T],$$

where $p_0 \in L^1[0, T]$. Then there are σ_0 (depending on p_0) and φ such that for each $\varepsilon > 0$ there exists a solution $y \in W^{4,1}$ of the above problem such that almost everywhere in $t \in [c, d]$ we have $|\mathcal{D}y(t) - \mathcal{D}y_0(t)| \le \sigma_0(t)$,

$$|y(t) - y_0(t)| \le (\varphi *_c \sigma_0)(t), \qquad |y'(t) - y_0'(t)| \le (\varphi' *_c \sigma_0 t)(t),$$
$$|y''(t) - y_0''(t)| \le (\varphi'' *_c \sigma_0)(t) \qquad |y'''(t) - y_0'''(t)| \le (\varphi''' *_c \sigma_0)(t),$$

where $*_c$ stands for the convolution started at c.

Our estimates are constructive and more precise then those in the known versions of Filippov Lemma.

2010 *Mathematics Subject Classification.* 26A24, 28A15, 46G05, 39A05, 28A05.
Key words and phrases. Differential inclusion; beam differential operator; cosine family (of operators); Lipschitz multifunction; Filippov Lemma; Filippov–Ważewski Theorem.