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CONCENTRATION–COMPACTNESS FOR SINGULAR NONLOCAL SCHRÖDINGER EQUATIONS WITH OSCILLATORY NONLINEARITIES

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Abstract. The paper is dedicated to the theory of concentration–compactness principles for inhomogeneous fractional Sobolev spaces. This subject for the local case has been studied since the publication of the celebrated works due to P.-L. Lions, which laid the broad foundations of the method and outlined a wide scope of its applications. Our study is based on the analysis of the profile decomposition for the weak convergence following the approach of dislocation spaces, introduced by K. Tintarev and K.-H. Fieseler. As an application, we obtain existence of nontrivial and nonnegative solutions and ground states for fractional Schrödinger equations for a wide class of possible singular potentials, not necessarily bounded away from zero. We consider possible oscillatory nonlinearities for both cases, subcritical and critical which are superlinear at the origin, without the classical Ambrosetti and Rabinowitz growth condition. In some of our results we prove existence of solutions by means of compactness of Palais–Smale sequences of the associated functional at the mountain pass level. To this end we study and provide the behavior of the weak profile decomposition convergence under the related functionals. Moreover, we use a Pohozaev type identity in our argument to compare the minimax levels of the energy functional with the ones of the associated limit problem. Motivated by this fact, in our work we also prove that this kind of identities hold for a larger class of potentials and nonlinearities for the fractional framework.

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