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INTEGRABILITY OF THE DERIVATIVE OF SOLUTIONS TO A SINGULAR ONE-DIMENSIONAL PARABOLIC PROBLEM

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To the memory of Professor Marek Burnat

Abstract. We study integrability of the derivative of a solution to a singular one-dimensional parabolic equation with initial data in $W^{1,1}$. In order to avoid additional difficulties we consider only the periodic boundary conditions. The problem we study is a gradient flow of a convex, linear growth variational functional. We also prove a similar result for the elliptic companion problem, i.e. the time semidiscretization.

1. Introduction

We study a one-dimensional parabolic equation

$$\begin{aligned} &u_t = (W_p(u_x))_x, \quad (x,t) \in Q_T := \mathbb{T} \times (0,T), \\ &u(x,0) = u_0(x), \quad x \in \mathbb{T}, \end{aligned} \tag{1.1}$$

where $W : \mathbb{R} \to \mathbb{R}$, $W_p = dW(p)/dp$ and $\mathbb{T}$ is a flat one-dimensional torus, which we identify with $[0,1)$. In other words, for the sake of simplicity we consider the periodic boundary conditions, but the same argument with little change applies to the zero Neumann data.

Equation (1.1) is formally a gradient flow of the following functional,

$$\mathcal{E}(u) = \begin{cases} \displaystyle\int_{\mathbb{T}} W(u_x)\,dx & \text{for } u \in W^{1,1}(\mathbb{T}), \\ +\infty & \text{for } u \in L^2(\mathbb{T}) \setminus W^{1,1}(\mathbb{T}). \end{cases}$$

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