Topological Methods in Nonlinear Analysis
Volume 52, No. 1, 2018, 195–212
DOI: 10.12775/TMNA.2017.049

A NOTE ON THE 3-D NAVIER–STOKES EQUATIONS

Jan W. Cholewa — Tomasz Dłotko

In memory of Professor Marek Burnat

Abstract. We consider the Navier–Stokes model in a bounded smooth domain $\Omega \subset \mathbb{R}^3$. Assuming a smallness condition on the external force f, which does not necessitate smallness of $\|f\|_{[L^2(\Omega)]^3}$-norm, we show that for any smooth divergence free initial data u_0 there exists $\mathcal{T} = \mathcal{T}(\|u_0\|_{[L^2(\Omega)]^3})$ satisfying

$$\mathcal{T} \to 0 \quad \text{as } \|u_0\|_{[L^2(\Omega)]^3} \to 0 \quad \text{and} \quad \mathcal{T} \to \infty \quad \text{as } \|u_0\|_{[L^2(\Omega)]^3} \to \infty,$$

and such that either a corresponding regular solution ceases to exist until $\mathcal{T}$ or, otherwise, it is globally defined and approaches a maximal compact invariant set $\mathbb{A}$. The latter set $\mathbb{A}$ is a global attractor for the semigroup restricted to initial velocities u_0 in a certain ball of fractional power space $X^{1/4}$ associated with the Stokes operator, which in turn does not necessitate smallness of the gradient norm $\|\nabla u_0\|_{[L^2(\Omega)]^3}$. Moreover, $\mathbb{A}$ attracts orbits of bounded sets in X through Leray–Hopf type solutions obtained as limits of viscous parabolic approximations.

2010 *Mathematics Subject Classification.* 35Q30, 35B40.

Key words and phrases. Navier–Stokes equations; global solutions; small data; asymptotic behavior.

The first named author is supported by grant MTM2016-75465 from MINECO, Spain.