THREE-DIMENSIONAL THERMO-VISCO-ELASTICITY
WITH THE EINSTEIN–DEBYE \((\theta^3 + \theta)\)-LAW
FOR THE SPECIFIC HEAT.
GLOBAL REGULAR SOLVABILITY

IRENA PAWŁOW — WOJIECH M. ZAJĄCZKOWSKI

Abstract. A three-dimensional thermo-visco-elastic system for the
Kelvin–Voigt type material at small strain is considered. The system in-
volves the constant heat conductivity and the specific heat satisfying the
Einstein–Debye \((\theta^3 + \theta)\)-law. Such a nonlinear law, relevant at relatively low
temperatures, represents the main novelty of the paper. The existence
of global regular solutions is proved without the small data assumption. The
crucial part of the proof is the strictly positive lower bound on the absolute
temperature \(\theta\). The problem remains open in the case of the Debye \(\theta^3\)-law.
The existence of local in time solutions is proved by the Banach successive
approximations method. The global \textit{a priori} estimates are derived with the
help of the theory of anisotropic Sobolev spaces with a mixed norm. Such
estimates allow to extend the local solution step by step in time.

1. Introduction

The aim. In this paper we study the three-dimensional (3-D) thermo-visco-
elastic system at small strains with the constant heat conductivity \(k > 0\), and
the specific heat (heat capacity) \(c(\theta)\) satisfying the Einstein–Debye \((\theta^3 + \theta)\)-law,

2010 Mathematics Subject Classification. Primary: 74R90, 74K50; Secondary: 74Q71, 74F05.

Key words and phrases. Thermo-visco-elastic system; Kelvin–Voigt materials; Einstein–
Debye law for specific heat; Sobolev spaces with a mixed norm; existence of global regular
solutions.