Topological **M**ethods in **N**onlinear **A**nalysis
Volume 52, No. 1, 2018, 161–193
DOI: 10.12775/TMNA.2018.016

THREE-DIMENSIONAL THERMO-VISCO-ELASTICITY WITH THE EINSTEIN–DEBYE $(\theta^3+\theta)$-LAW FOR THE SPECIFIC HEAT. GLOBAL REGULAR SOLVABILITY

Irena Pawłow — Wojciech M. Zajączkowski

We dedicate the paper to Professor Marek Burnat for his personal warmness and devotion to mathematics

Abstract. A three-dimensional thermo-visco-elastic system for the Kelvin–Voigt type material at small strain is considered. The system involves the constant heat conductivity and the specific heat satisfying the Einstein–Debye $(\theta^3+\theta)$-law. Such a nonlinear law, relevant at relatively low temperatures, represents the main novelty of the paper. The existence of global regular solutions is proved without the small data assumption. The crucial part of the proof is the strictly positive lower bound on the absolute temperature θ. The problem remains open in the case of the Debye θ^3-law. The existence of local in time solutions is proved by the Banach successive approximations method. The global *a priori* estimates are derived with the help of the theory of anisotropic Sobolev spaces with a mixed norm. Such estimates allow to extend the local solution step by step in time.

1. Introduction

The aim. In this paper we study the three-dimensional (3-D) thermo–visco-elastic system at small strains with the constant heat conductivity $k>0$, and the specific heat (heat capacity) $c(\theta)$ satisfying the Einstein–Debye $(\theta^3+\theta)$-law,

2010 *Mathematics Subject Classification.* Primary: 74B20, 35K50; Secondary: 35Q71, 74F05.

Key words and phrases. Thermo-visco-elastic system; Kelvin–Voigt materials; Einstein–Debye law for specific heat; Sobolev spaces with a mixed norm; existence of global regular solutions.