BLOWUP VERSUS GLOBAL IN TIME EXISTENCE OF SOLUTIONS FOR NONLINEAR HEAT EQUATIONS

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In memory of Marek Burnat

ABSTRACT. This note is devoted to a simple proof of blowup of solutions for a nonlinear heat equation. The criterion for a blowup is expressed in terms of a Morrey space norm and is in a sense complementary to conditions guaranteeing the global in time existence of solutions. The method goes back to H. Fujita and extends to other nonlinear parabolic equations.

1. Introduction

In this paper we consider the Cauchy problem for the simplest example of a semilinear parabolic equation in $\mathbb{R}^d$, $d \geq 1$, $p > 1$,

$$(1.1) \qquad u_t = \Delta u + |u|^{p-1}u, \quad x \in \mathbb{R}^d, \ t > 0,$$

$$(1.2) \qquad u(x,0) = u_0(x).$$

This problem has been thoroughly studied beginning with [18], [20], [21], and many fine properties of its solutions are known. For the reference, see the extensive monograph [32] and a recent paper [33].