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SCHAUDER'S THEOREM AND THE METHOD OF A PRIORI BOUNDS

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This article is dedicated to the memory of Professeur Marek Burnat

Abstract. We first recall simple proofs relying on the Schauder Fixed Point Theorem of the Nonlinear Alternative, the Leray–Schauder Alternative and the Coincidence Alternative for compact maps on normed spaces. We present also an alternative for compact maps defined on convex subsets of normed spaces. Those alternatives permit to apply the method of a priori bounds to obtain results establishing the existence of solutions to differential equations. Using those alternatives, we present some new proofs of existence results for first order differential equations.

1. Introduction

In this note, we first recall simple proofs relying on the Schauder Fixed Point Theorem of the Nonlinear Alternative, the Leray–Schauder Alternative and the Coincidence Alternative for compact maps on normed spaces [4], [5]. The avantage of those proofs is that they avoid the use of more sophisticated theories such as the topological degree theory, the topological transversality theory or the coincidence degree theory due to Mawhin [9]. To our knowledge, the first result in this direction was obtained by Schaefer [11].

In Section 2, we present some examples of applications of those alternatives with the method of a priori bounds to differential equations. First, we recall a generalization of a theorem due to S. Bernstein for second order differential

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