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ON CERTAIN VARIANT OF STRONGLY NONLINEAR MULTIDIMENSIONAL INTERPOLATION INEQUALITY

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Dedicated to the memory of Professor Marek Burnat

Abstract. We obtain the inequality

$$\int_\Omega |\nabla u(x)|^p h(u(x))\,dx \le C(n,p)\int_\Omega \Big(\sqrt{|\nabla^{(2)}u(x)||\mathcal{T}_{h,C}(u(x))|}\Big)^p h(u(x))\,dx,$$

where $\Omega\subset\mathbb{R}^n$ and $n\ge 2$, $u\colon\Omega\to\mathbb{R}$ is in certain subset in second order Sobolev space $W^{2,1}_{\rm loc}(\Omega)$, $\nabla^{(2)}u$ is the Hessian matrix of u, $\mathcal{T}_{h,C}(u)$ is a certain transformation of the continuous function $h(\,\cdot\,)$. Such inequality is the generalization of a similar inequality holding in one dimension, obtained earlier by second author and Peszek.

1. Introduction

The purpose of this paper is to obtain the n-dimensional variant of the following inequality:

$$\int_{(a,b)} |u'(x)|^p h(u(x))\,dx \le C_p \int_{(a,b)} \Big(\sqrt{|u''(x)\mathcal{T}_h(u(x))\|}\Big)^p h(u(x))\,dx, \tag{1.1}$$

where (a,b) is an interval, $u\in W^{2,1}_{\rm loc}((a,b))$ and obeys some additional assumptions, $h(\,\cdot\,)$ the a given continuous function, $\mathcal{T}_h(\,\cdot\,)$ is a certain transform of $h(\,\cdot\,)$.

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