

ON TWO SYMMETRIES IN THE THEORY OF m -HESSIAN OPERATORS

NINA M. IVOCHKINA AND NADEZHDA V. FILIMONENKOVA

Dedicated to the memory of Marek Burnat

ABSTRACT. The modern theory of fully nonlinear operators had been inspired by the skew symmetry of minors in cooperation with the symmetry of symmetric functions. We present some consequences of this interaction for m -Hessian operators. One of them is setting of the isoperimetric variational problem for Hessian integrals. The m -admissible minimizer is found that allows a new simple proof of the well-known Poincaré-type inequalities for Hessian integrals. Also a new set of inequalities, generated by a special finite set of functions, is presented.

1. Introduction

The modern theory of fully nonlinear second-order partial differential equations counts more than 35 years and has been initiated in the papers [8], [19], where the a priori estimates of Hölder constants for the second derivatives of solutions have been established. It reduced the problem of classical solvability of the Dirichlet problem for fully nonlinear second-order partial differential equations to finding the a priori estimate of solutions in C^2 . For an attempt to give a general description of obtaining this estimate for fully nonlinear operators we refer to [3], [4], [19].

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